Typicality: An Improved Semantic Analysis

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Abstract

Parts 1-3 present and criticize Partee and Kamp's 1995 well known analysis of the typicality effects. The main virtue of this analysis is in the use of supermodels, rather than fuzzy models, in order to represent vagueness in predicate meaning. The main problem is that typicality of an item in a predicate is represented by a value assigned by a measure function, indicating the proportion of supervaluations in which the item falls under the predicate. A number of issues cannot be correctly represented by the measure function, including the typicality effects in sharp predicates; the conjunction fallacy; the context dependency of the typicality effects etc. In Parts 4-5, it is argued that these classical problems are solved if the typicality ordering is taken to be the order in which entities are learnt to be denotation members (or non-members) through contexts and their extensions. A modified formal model is presented, which clarifies the connections between the typicality effects, predicate meaning, and its acquisition.

Contents:

1. What are the typicality effects?

Speakers order entities or sub-kinds (Dayal 2004; sub-kinds are also called *exemplars*) by their typicality in predicates. For example, a robin is often considered *more typical of a bird* than an ostrich or a penguin.

These ordering judgments show up in an unconcious processing effect, namely in *online categorization time*: Verification time for sentences like *a robin is a bird*, where subjects determine category membership for a typical item, is faster than for sentences like *an ostrich is a bird*, where subjects determine membership of an atypical item (Rosch 1973, Armstrong, Gleitman and Gleitman 1983).

In addition, speakers consider features like *feathers, small, flies* and *sings*, as *typical of birds*. Crucially, the more typical birds are more typical in these features (Rosch 1973).

These judgments are highly context dependent. For example, within a context of an utterance like: *the bird walked across the barnyard*, a chicken is regarded as a typical bird, and categorization time is faster for the contextually appropriate item *chicken* than for the normally typical but contextually inappropriate item *robin* (Roth and Shoben 1983).

In addition to these basic effects, there are robust *order of learning effects*. In a nutshell, typical instances are acquired earlier than atypical ones, by children of various ages and by adults (Mervis and Rosch 1981, Rosch 1973, Murphy and Smith 1982); in recall tasks, typical instances are produced before atypical ones (Rosch 1973, Batting & Montague 1969); categories are learned faster if initial exposure is to a typical member (Mervis & Pani 1980), than if initial exposure is to an atypical member, or even to the whole denotation in a random order; and finally, typical (or early acquired) instances are remembered best (Heit 1997), and they affect future learning (encoding in memory) of entities and their features (Rips 1975, Osherson et al 1990). In sum, typicality is deeply related to the order in which instances are learnt to be members in predicate denotations.

These findings were replicated time and again (Mervis and Rosch 1981). Yet, *the mental models underlying them* and *their relation to predicate meaning* are still a puzzle. To see this, we will now review the well known typicality theory, which is most frequently cited by formal semanticists, namely – *The Supermodel Theory*. For a more detailed discussion of the typicality effects and other model types, see Sassoon 2005.

2. The Supermodel Theory (Partee and Kamp 1995)

2.1 Background: Multiple valued logic in the analysis of typicality

Partee and Kamp's main innovation within the analysis of typicality, is in the use of a logic *with three truth values* and the technique of *Supervaluations* (van Fraassen 1969; Kamp 1975; Fein 1975; Veltman 1984; Landman 1991), as opposed to the standard use of a logic with *multiple truth values* (such as *fuzzy logics*) in the analysis of typicality in artificial intelligence, cognitive psychology, and linguistics (Zadeh 1965; Lakoff 1973; Osherson & Smith 1981; Lakoff 1987; Aarts et al 2004).

2.1.1 Fuzzy models

In classical logics, a proposition may take as a truth value either 0 or 1. In fuzzy logics, a proposition may take as a truth value any number in the real interval [0,1]. For example, such a model can assume the following facts:

[1] The truth value of the proposition *a robin is a bird* is 1;
The truth value of the proposition *a goose is a bird* is 0.7;
The truth value of the proposition *an ostrich is a bird* is 0.5;
The truth value of the proposition *a butterfly is a bird* is 0.3;
The truth value of the proposition *a cow is a bird* is 0.1.

These values indicate the typicality degrees of the individuals or kinds denoted by the subjects in the predicate *bird*.

More precisely, in such models, predicates are not associated with sets as denotations. Rather, for every predicate P, *a characteristic function*, $\mathbf{c}_{\mathbf{m(P,d)}}$, assigns to each entity d in the domain of individuals D, a value in the real interval [0,1], *its degree of membership in P*. Moreover, each predicate is associated with *a prototype* **p**, i.e. the best member possible. Finally, a *degree function* $\mathbf{c}_{\mathbf{P}}$ (a distance metric) associates pairs of entities with values in the real interval [0,1]. If, for example, r is a robin, b a blue jay and o an ostrich, then: $\mathbf{c}_{\mathbf{P}}(r,b) < \mathbf{c}_{\mathbf{P}}(r,o)$, i.e. *r is more similar to b than to o*. The typicality of an entity d in P is represented as the distance of d from the prototype of P, $\mathbf{c}_{\mathbf{P(d,p)}}$. This distance function satisfies several constraints. For example, $\mathbf{c}_{\mathbf{P}}$ is such that any entity has zero distance from itself ($\forall d \in D$: $\mathbf{c}_{\mathbf{P}(d,d)} = 0$); $\mathbf{c}_{\mathbf{P}}$ is symmetric ($\forall d,e \in D$: $\mathbf{c}_{\mathbf{P}(d,e)} = \mathbf{c}_{\mathbf{P}(e,d)}$); and $\mathbf{c}_{\mathbf{P}}$ has the property called *the triangle inequality* ($\forall d,e,f \in D$: $\mathbf{c}_{\mathbf{P}(d,e)} + \mathbf{c}_{\mathbf{P}(e,f)} \geq \mathbf{c}_{\mathbf{P}(d,f)}$). Most important for our purposes is the *monotonic decreasing relation* between d and c: The distance of entities from the prototype p of P inversely correlates with their membership degree in P:

[2] $\forall d,e \in D$: $(\mathbf{c}_{\mathbf{P}(d,p)} \leq \mathbf{c}_{\mathbf{P}(e,p)}) \rightarrow (c_{m(P,d)} \geq c_{m(P,e)})$.

Typicality degrees are assumed to correspond to degrees, or probabilities, of membership in the category. This leading intuition shows up also in the rules that predict the typicality degrees in complex predicates. There are three composition rules for c_m:

[3] 1. The *complement rule* for $\neg$: $c_{m(\neg P,d)} = 1 - c_{m(P,d)}$
2. The *minimal-degree rule* for $\wedge$: $c_{m(P \wedge Q,d)} = \mathrm{Min}(c_{m(P,d)}, c_{m(Q,d)})$
3. The *maximal-degree rule* for $\vee$: $c_{m(P \vee Q,d)} = \mathrm{Max}(c_{m(P,d)}, c_{m(Q,d)})$

Consider, for instance, the *complement rule* for negated predicates in (3.1). The degree of a *goose* in *not-a-bird* is assumed to be the complement of its degree in *bird* (e.g. 1- 0.7). This rule is directly inspired by the idea that the probability that *p* is the complement of the probably that *not-p*.

Similarly, the *minimal-degree rule* for conjunctions in (3.2) states that an item's degree in a modified noun like *brown apple* is the minimal degree among the constituents, *brown* and *apple*. This rule, and other versions of the rule for conjunctions and modified nouns in fuzzy models, are directly inspired by the fact that the probability that $p \wedge q$ cannot exceed the probability that just *p*, or just *q*.

2.1.2 Problems of fuzzy models

Osherson and Smith 1981 have shown a variety of shortcomings of fuzzy models. Following them, Partee and Kamp 1995 have argued at length against such models. The main problem for these models is that they generate wrong predictions.

Consider, for example, *the-minimal-degree rule*. This rule predicts that the typicality degree of, e.g. brown-apples, cannot be bigger in *brown apple* than in *apple*. Hence, this rule fails to predict the empirically well established *conjunction effect* (Smith et al 1988) or *fallacy* (Tversky et al 1983), i.e. the finding that, according to speakers' intuitive judgments, both the typicality degree (Smith et al 1988), and the likelihood of category membership (Tversky et al 1983), of brown-apples, is bigger in *brown apple* than in *apple*.

The minimal-degree rule is most problematic when it comes to *contradictory* and *tautological predicates*. Intuitively, the degree of all entities in $P\wedge\neg P$ and $P\vee\neg P$ ought to be 0 and 1, respectively. But fuzzy models fail to predict this. For example, if *a goose is a bird* to degree 0.7, then according to the complement rule, *a goose is not a bird* to degree 0.3. Given this, the minimal degree rule predicts that *a goose is a bird and not a bird* to degree 0.3, rather than to degree 0.

Another problem has to do with the fact that the degree function in these models is total, though knowledge about typicality is often partial. For example, if one bird sings and the other flies, which one is more typical? We cannot tell out of context. This problem highlights the need for more context dependency in the representation of typicality. Partee and Kamp 1995 have argued at length for the importance of this aspect. Yet, we will see in part 3 that their proposal is also insufficient in this respect.

A problem which usually goes unnoticed has to do with the complement rule. It is indeed true that the typicality orderings of negated predicates are essentially the reverse of the orderings of the predicates that are being negated (see, for instance, the findings reported in Smith et al 1988), yet exceptions to this rule are quite common. Why? Because negated predicates are often contextually restricted. For example, the set of *non-birds* is frequently assumed to only consist of *animals*. In such contexts, *non-animals* are intuitively assigned low typicality degrees both in the predicate *bird* and in the negated predicate *non-bird* (rather than a low degree in *bird* and a high degree in *non-bird*, as predicted by the complement rule). This judgment is not captured because the relevant contextual factors are not represented.

2.1.3 Intermediate summary

We saw that multiple truth values, or probability degrees, as means to indicate typicality degrees, are problematic in many respects. An alternative theory is the *Supermodel Theory* (Partee and Kamp 1995). This analysis uses the same types of mechanisms, namely – a membership degree function $\mathbf{c_m}$, a prototype **p**, and a typicality degree function $\mathbf{c_p}$. However, this analysis differs in two crucial respects. First, it replaces fuzzy logics with three valued logics. Second, the typicality degrees are not always coupled with the membership degrees. With these two differences, the analysis is claimed to be significantly improved. However, while indeed improved in some respects, we will see in part 3 that this analysis is highly limited and problematic in other respects. In part 4 we will propose a novel analysis which completely abandons the use of membership degree functions, prototypes, and distance functions.

2.2 Supermodels

A supermodel M^* consists of one partial model M, which I will call 'context' M. In M, denotations are only partially known. For example, the denotation of *chair* in a partial context M may consist of only one item – the prototypical chair, p_{chair}. The denotation of *non-chair* may consist of only one item too, which is very clearly not a chair, say – the prototypical sofa, p_{sofa}. This means that in M we don't yet know if anything else, (an armchair, a stool, a chair with less than 4 legs, a chair without a back, a chair which is not used as a seat, a chair which is not of the normal size etc.), is a *chair* or not.

In addition, M is accompanied by a set T of total models (the *supervaluations* in van Fraassen 1969), i.e. a set of all the possibilities seen in M to specify the complete sets of *chairs* and *non-chairs*. In each t in T, each item is either in the denotation of *chair* or in the denotation of *non-chair*.

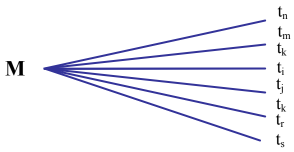


Figure 1: The context structure in a supermodel M^*

Formally, a supermodel M^* for a set of predicates A and a set of entities D is a tuple **<M,T,m>** such that:

[1] **M** is a *partial model*: Predicates are associated with partial denotations in M, **$<[P]^+_M,[P]^-_M>$**.
For example, if $[chair]^+_M = \{d_1\}$, $[chair]^-_M = \{d_3\}$, d_2 is in the gap, we don't yet know if it is a chair or not.

[2] **T** is *a set of total models* which are completions of M:
Predicates are associated with total denotations, which are monotonic extentions of their denotations in M:
$\forall t \in T, \forall P \in A$:
2.1. **Maximality**: $[P]^+_t \cup [P]^-_t = D$ (denotations are total).
2.2. **Monotonicity**: $[P]^+_M \subseteq [P]^+_t$; $[P]^-_M \subseteq [P]^-_t$.
E.g. in each $t \in T$, d_2 is added to $[chair]^+_t$ or $[chair]^-_t$.

Given this basic ontology, the *membership degree* of an individual d in a vague noun like *chair* is indicated by the size or *measure* of the set of total contexts in which d is a *chair*, **m(**{t∈T: d∈$[chair]^{+}_{t}$}).

For example, the prototypical chair, p_{chair}, is a *chair* in all total possibilities, so its membership degree is 1. The prototypical sofa, p_{sofa}, is a *chair* in no possibility, so its membership degree is 0. If an armchair d is a *chair* in a third of the cases, its membership degree is 1/3 etc.:

[3] m is *a measure function* from sets of total models to real numbers between 0 and 1, i.e. a function which satisfies the following constraints (Partee and Kamp 1995, p. 153):

3.1 m(T) = 1;

3.2 m({}) = 0;

3.3 $\forall T_1,T_2$, s.t. $T_1 \subset T_2$: $m(T_2) = m(T_1) + m(T_2 - T_1)$ etc.

[4] The *membership-degree* of d in P, $\mathbf{c_{m(d,P)}}$, is given by the measure m of the set of total models in which d is P:

$c_{m(d,P)} = m(\{t \in T: d \in [P]^{+}_{t}\})$

e.g. $1 = c_{m(d1,chair)} > c_{m(d2,chair)} > c_{m(d3,chair)} = 0$.

There is no doubt that this model is better suited to the representation of natural language than fuzzy models. For example, we now predict membership degrees 0 and 1 in contradictory and tautological predicates respectively, as opposed to the prediction of the minimal degree rule in fuzzy models (cf. 2.1). This is because for all total contexts t in T, it holds that no entity falls under $P\wedge\neg P$, and all entities fall under $P\vee\neg P$. Thus, even if, say, a certain stool is *a chair* to degree 0.7 and *not a chair* to degree 0.3 (due to being regarded as a *chair* in 0.7 of the total contexts in T, and being regarded as a *non-chair* in the rest of T), it is *a chair and not a chair* to degree 0, and *a chair or not a chair* to degree 1.

2.3 The representation of typicality in the Supermodel Theory

2.3.1 Typicality in basic predicates

In this theory, a degree of membership and a degree of typicality are taken to be two separate things. The typicality degree of an entity in a predicate is represented by the entity's similarity to (or distance from) the predicate's prototype. Typicality and membership are assumed to be coupled only in vague nouns like *chair*. In sharp nouns like *bird* or *grandmother,* they may be dissociated. Thus:

[5] A predicate P is associated with a tuple **<p, c_m, c_P>** such that:

1. **p** is the prototype – the best possible P.
2. $\mathbf{c_{m(d,P)}}$**,** is d's *membership-degree* in P: the degree to which d is P. As explained in 2.2, it is given by the measure m of the set of total contexts in which d is a *chair*: $c_{m(d,P)} = m(\{t \in T: d \in [P]^+_t\}$.
3. $\mathbf{c_{P(d,P)}}$ is d's *typicality-degree* in P: d's distance from P's prototype.

How are the values of the typicality degree function, $c_{P(d,P)}$, indicated? Generally, they are given by the values of the membership function: $c_P \cong c_m$: e.g. in *chair*: the more typical entities fall under $[chair]^+$ in more of the total models t in T. However, Partee and Kamp distinguish between different predicate types in the following ways:

[6] Predicate types:

1. **+/– Vague**:
   The denotations of non-vague predicates like *bird*, unlike those of vague predicates like *chair*, are total already in M. That is, everything is either a *bird* or a *non-bird*. There is no gap: $[bird]^+_M \cup [bird]^-_M = D$.
2. **+/– Prototype**:
   Predicates like *tall* or *odd number*, unlike *bird, grandmother, red* etc., have no prototype (because there is no maximal *tallness* or *oddness*).
3. **+/– Typicality-is-coupled-with-membership,** $\mathbf{c_P \cong c_m}$
   (The original term is: *+/–the-prototype-affects-the-denotation*): In predicates like *bird* or *grandmother*, unlike predicates like *chair*, typicality and membership are separated (not coupled).

	–Prototype	**+Prototype**	
		($c_m \neq c_P$)	**($c_m = c_P$)**
+Vague	*tall, wide, heavy, not red*	*adolescent, tall tree*	*red, chair, shy*
–Vague	*even, odd, inanimate, not a bird*	*bird, grandmother*	∅

Table 1: Predicate types in Partee and Kamp's analysis

There are at least two reasons for the separation of typicality and membership in predicates like *bird*:

(1) Intuitively, an *ostrich* d is a *bird* even in M, i.e. $c_{m(d,bird)} = 1$; but it is an atypical *bird*, i.e. $c_{P(d,bird)} < 1$. Thus, $c_m \neq c_P$.

(2) Intuitively, an *ostrich* is always a *bird,* i.e. for any entity d, the set of total contexts in which d is an *ostrich*, $\{t\in T: d\in[ostrich]^+_t\}$, is always a subset of the set of total contexts in which d is a *bird*, $\{t\in T: d\in[bird]^+_t\}$. So $c_{m(d,ostrich)}$ is always smaller than $c_{m(d,bird)}$:

$$c_{m(d,\ ostrich)} = m(\{t\in T: d\in[\ ostrich]^+_t\}) \leq m(\{t\in T: d\in[\ bird]^+_t\}) = c_{m(d,\ bird)}$$

But intuitively, d can be *more typical* of *an ostrich* than of a *bird*, so $c_{P(d,ostrich)}$ is greater than $c_{P(d,bird)}$.

$$c_{P(d,\ ostrich)} \geq c_{P(d,\ bird)}.$$

Again, $c_m \neq c_p$.

Let us classify the fact that d can be *more typical* of *an ostrich* than of a *bird*, as stated in (2), under the name *the sub-type effect* (Sassoon 2005).

2.3.2 Typicality in complex predicates

Recall the *conjunction effect* or *fallacy*, i.e., the intuitive judgments that, e.g., a brown-apple is regarded as *more typical*, or *more likely a member*, in *brown apple* than in *apple* (see in *2.1.2*):

$$c_{P(d,\ brown\ apple)} \geq c_{P(d,\ apple)}.$$

This effect cannot be represented using Partee and Kamp's membership degree function $c_{m(d,P)}$. Why? Because in any total context in which an entity d is a *brown apple,* d is an *apple*, i.e. the set $\{t\in T: d\in[\ brown\ apple]^+_t\}$ is always a subset of the set $\{t\in T: d\in[apple]^+_t\}$. Hence, the membership degree of d in *brown apple* can maximally reach d's degree in *apple* and not more:

$$c_{m(d,brown\ apple)} = m(\{t\in T: d\in[brown\ apple]^+_t\}) \leq m(\{t\in T: d\in[apple]^+_t\}) = c_{m(d2,apple)}$$

However, Partee and Kamp observe that modifiers like *brown* receive a distinct interpretation in each of the local contexts created by the noun they modify. For example, *brown* is interpreted differently when applied to *apple*, *skin, shelf, dress* etc. Thus, Partee and Kamp propose to replace c_m in modified nouns like *brown apple* by a new function, which may assign d a higher value than $c_{m(d,apple)}$ or $c_{m(d,brown)}$. *The modified membership function* for the modified noun *brown apple*, $\mathbf{c}_{\mathbf{m(d,}brown\,/apple\mathbf{)}}$ is given by d's degree in *brown*, $m_{(d,brown)}$, minus 'a' – the minimal *brown* degree that the measure function m assigns to an apple. This value is normalized by the distance between 'a' - the minimal - and 'b' - the maximal - *brown* degrees assigned to apples. This normalization procedure ensures that the result ranges between 0 and 1:

[7] *The modified membership function for modified nouns:*
Let a and b be the minimal and maximal *brown* degrees among the apples in M, respectively: $\mathbf{c}_{\mathbf{m(d,}brown\,/apple\mathbf{)}} = (m_{(d,brown)} - a) / (b - a)$

For example, a brown apple may be assigned degree 0.9 in *brown*; the minimal *brown* degree existing among the apples may be 0, because some apples are not *brown* at all; the maximal *brown* degree existing among the apples may be 0.95, assuming that no apple is maximally brown. If so:

$$\mathbf{c}_{\mathbf{m}(d,brown\,/apple)} = (0.9 - 0) / (0.95 - 0) = 0.974.$$

The value 0.974 indeed exceeds d's degree in *brown*, 0.9, and possibly also d's degree in *apple*, as desired. If indeed, the proposed mechanism helps to capture the conjunction fallacy, it seems like we could retain the idea that the typicality degrees in predicates like *brown apples* are coupled with the membership degrees, which in turn, are indicated by the modified membership functions. However, we will now see that this is not the case.

3. Problems in the Supermodel Theory

The idea that measures-functions which range over total contexts (supervaluations) can represent typicality has some fundamental problems.

3.1 Typicality degrees of denotation members

The first problem has to do with the fact that the measure function m fails to account for the fact that denotation members are not necessarily associated with the maximal degree of typicality, 1, but rather they may take any degree of a whole range of typicality degrees. For example, within a certain context, I may consider three-legged seats with a back as *chairs*, but as *less typical chairs* than four-legged seats with a back.

This limitation of the measure function is particularly problematic in vague nouns (sharp nouns) like *bird*. Even atypical examples like *ostriches* and *penguins* are known to be *birds*, i.e. already in M they are considered members in $[bird]^{+}_{M}$ (Partee and Kamp 1995). The *bird* denotations are assumed to be completely specified, or in other words, not to vary across different total contexts. This is the standard way in which to represent the fact that predicates like *bird* are not – or are much less – vague than predicates like *chair* or *tall*. However, this is also the reason for which the measure function cannot indicate typicality in sharp predicates. Given that they are always known to be birds, the membership degree of atypical examples like *ostriches* and *penguins* in *bird* (i.e. the measure of the set of total contexts in which they are *birds*) is always 1. And for non-birds – whether *butterflies* and *bats* or whether *stools* and *cows* – since they are members in $[bird]^{-}_{M}$, their membership degree in *bird* is always 0. Intermediate typicality degrees in sharp nouns cannot be indicated using m. Since no other means to indicate them is given, i.e. no general mechanism to determine distance from the prototype is proposed, intermediate typicality degrees in sharp nouns are not accounted for.

This is especially problematic given that the most prominent examples of the prototype theory are indeed sharp predicates.

3.2 The sub-type effect

Furthermore, the measure function, m, fails to predict *the sub-type effect*, namely, the intuition that the typicality of ostriches in *ostrich* exceeds their typicality in *bird*. A membership degree (or measure m) is never bigger in *ostrich* than in *bird*, because in any total context in which an entity is an *ostrich*, it is also a *bird* (see *2.3.1*). This effect is identical to the so-called *conjunction effect,* but is found in lexical nouns, i.e. nouns without a modifier, like *ostrich* vs. *bird*.

Note that the modified membership function, which Partee and Kamp add to the model in order to capture the *conjunction fallacy / effect* (see *2.3.2*), cannot help us here. Why? Because the minimal and maximal *ostrich* degrees in $[bird]^{+}_{M}$ are 0 and 1. We can find both complete ostriches (of membership degree 1) and complete non-ostriches (of membership degree 0) among the birds. Consequently, $c_{m(d,ostrich\,/\,bird)}$ is identical to $c_{m(d,ostrich)}$:

$$c_{m(d,ostrich\,/\,bird)} = \quad (m_{(d,ostrich)} - 0) / (1\text{-}0) \quad = \quad c_{m(d,ostrich)}$$

Thus, we have to keep c_m and c_P separated in such lexical nouns. It is the values of c_P which represent the intermediate typicality degrees and the sub-type effect / fallacy in *bird.* But, again, Partee and Kamp do not specify how exactly the values of c_P are determined when c_m and c_P are dissociated. Thus, the sub-type effect in lexical nouns is not accounted for, and in addition to this, the separation between c_m and c_p (in predicates like *bird*) forces us into an inelegant theory, which stipulates as primitives two unconnected sets of values for c_m and c_p.

Finally, the typicality effects in basic and complex nouns are accounted for using separate measure functions (given in **[5]** in *2.3.1* and **[7]** *in 2.3.2*). But we would prefer an account using a single mechanism, given that certain complex nouns in English are basic lexical items in other languages. For example, *'male-nurse*' translates into the basic noun *ax* in Hebrew.

3.3 The conjunction effect

Worse still, conjunction fallacies in modified nouns are also not dealt with correctly (see *2.3.2*). Indeed, brown apples are allowed to have greater degrees in *brown apple* than in *brown* or in *apple*, as desired, but they are ordered only by how *brown* they are. This yields incorrect degrees. Intuitively, an apple of an unusual shape or size, which is therefore assigned, say, typicality degree 0.2 in *apple*, even if maximally *brown* (of typicality and membership degree 1 in *brown*), is considered an atypical *brown apple,* and not a maximally typical *brown apple*, or a *brown apple* to degree 1, as predicted by Partee and Kamp's analysis:

$$c_{m(d,brown\,/apple)} = \quad (m_{(d,brown} - a) / (b - a) = \quad (1 - 0) / (1 - 0) = \quad 1$$

Thus, assuming that the typicality degrees in *brown apple* are assigned by the modified degree function, is incorrect. We have to assume that the typicality degrees in *brown apple* are assigned by another mechanism. For further empirical support to this argument, see Smith et al 1988.

There are many naturally occurring examples of utterances which refer to typicality in complex predicates. The following examples were found in a simple Google search on the Internet, and they contain references to typicality in negated and/or modified nouns:

1) *What were some exercises you would do* ***on a typical non-running day****? I read that they are mainly variations of pushups and situps, but what exactly are...*
2) *... there is one week where the format will be* ***more typical of a non-seminar class****...*
3) *Thought it [the interview] pretty mu****ch typical of a non-fan, non-entertainment, smart up market British paper*** *... it gives you some sense of being there and imagine what it's like to interview a 'star'.*
4) *You counter with an anecdotal tale about* ***a non-typical non-developer****. How does your counter-argument apply to* ***a typical non-developer****?*
5) *...her irritating non-performance is* ***typical of a primarily young (read 'cheap') cast…***
6) *The music is* ***typical of a non-CD game*** *- that is to say, worthless. It's tinny and very electronic sounding.*

Given these examples, we cannot dismiss the problems in predicting typicality in complex predicates on the basis that typicality is inherently non-compositional. Though compositionality might be limited to some extent, we need an analysis which will more correctly predict speakers' intuitions about typicality in complex predicates when such intuitions exist.

3.4 Partial knowledge

Thus far, we have focused on problems related to the representation of the typicality effects in sharp predicates and in complex predicates. Let us add to this picture now another classical problem concerning the representation of context dependency in the typicality judgments.

This problem has to do with the fact that the measure functions (or the membership functions) are total (in every partial model M, every entity is assigned a degree in every predicate), though knowledge about typicality is often partial. If one bird sings and the other flies, which one is more typical?

Which bird is more typical – an ostrich or a penguin? Many contexts are too partial to tell such facts. (Nor do speakers know every typicality feature in every partial context. For example is *in the home* typical of chairs?) The representation of knowledge about typicality needs to be more inherently context dependent and possibly partial.

One way to do this is to define the typicality function so that it will give each entity a value in a predicate *in each total context separately* (like the interpretation function). In such a way, it would be possible that the typicality degree of an entity (just like its membership in a predicate) is unknown in a partial model M. It would be unknown if and only if this entity's degree varies across different total contexts. However, note that the measure function in Partee and Kamp 1995 is defined per supermodel (it is a meaure of the proportion of valuations in T in which each item is a predicate member), so it is not easy to see how this measure function can be relativized to a total context.

3.5 Numerical degrees

Another problem common both to fuzzy models and to supermodels is that numerical degrees are not intuitive primitives. For example, why would a certain penguin have a degree 0.25 rather than say 0.242 in *bird*?

Partee and Kamp notice this problem and draw a general suggestion for a solution in terms of vagueness with regard to the correct measure function in each context. In this setting, a context is associated with a set of measure functions, such that we may only know in a certain context that, e.g., the degree of a penguin ranges between 0.25 to 0.242 in *bird*. Working this idea out would have been a step towards the addition of more context dependency into the representation (cf. 3.4!). However, Partee and Kamp admit that this is still complex and not quite a natural representation.

It is true that in the languages of the world the comparative form *more P than* (or *less P than*) is derived from the predicate form *P* (which is assumed to stand for the concept: *P to degree* μ) and not vice versa (Klein 1980; Kamp 1975). Nevertheless, conceptually, at least as far as typicality is concerned, representing the typicality ordering denoted by a typicality comparative (e.g. the intuition that penguins are *less typical than* ducks, which in turn are *less typical than* robins etc.), and deriving the degrees from this ordering by some general strategy (such that e.g. a penguin would have roughly zero typicality in *bird*) seems to be a more intuitive setting.

Arguments can be given also for a difference between the linguistic and conceptual setting in predicates and comparatives without the typicality operator (Fred Landman, personal communication), but these are beyond the scope of this paper.

3.6 Prototyopes

The notion of a prototyope is problematic in several respects.

One well known problem concerning this notion is that it is drastically unfruitful when it comes to compositionality, i.e., in predicting prototypes of complex concepts from the prototypes of their constituents (Partee and Kamp 1995; Hampton 1997). Consider negations: What would the prototype of *non-bird* be: a dog, a day, a number? Similarly for conjunctions: What would the *male-nurse* prototype be, given that a typical *male-nurse* may be both an *atypical male* and an *atypical nurse* (ibid).

Another problem has to do with predicates which are lacking a prototype. For example, there is no maximum tallness. But with no prototypes, the intuition that there are typical (and atypical) *tall players, tall teenagers, tall women* etc., is not accounted for. The status *prototypical*, so it seems, ought to be given to an entity only within a context (a valuation) – there are no context-independent entity-prototypes.

Finally, the Supermodel Theory assumes a complicated taxonomy of predicate types, with different mechanisms in their meaning (see Table 1 in *2.3.1*): With or without a prototype; with a prototype that affects the denotation or that does not affect the denotation; with a vague or a non-vague meaning etc. This is especially problematic when compositionality is addressed (Partee and Kamp 1995). For example, of what type are conjunctions of different predicate types, like *tall bird*, where *tall* is a vague predicate without a prototype, and *bird* is a non-vague predicate with a prototype?

3.7 Feature-sets

The main idea in assuming entity prototypes is to avoid the notion of *feature-sets*, which Partee and Kamp, following Osherson and Smith 1981 and Armstrong, Gleitman and Gleitman 1983, see as an ill-defined notion.

Back from Wittgenstein ([1953] 1968), feature-based models are most widespread in the analysis of typicality. Whether feature-sets are represented as frames (Smith et al 1988), networks (Murphy and Lassaline 1997), theories (Murphy and Medin 1985), vectors in conceptual spaces (Gardenfors 2004) or otherwise, the main idea is that each feature is assigned a weight.

The typicality degree of, say, a robin in *bird*, is indicated by *the weighted-mean of its degrees in the bird features*: How well it scores in *flies, sings* etc.

The problem is that features alone do not form a sufficient account. Scholars still hardly agree about how *the weight of a feature* is determined. Worse still, we can hardly tell how entities' *degrees in a feature* are determined. We still need to know ***what*** a typicality degree ***is*** (Armstrong, Gleitman and Gleitman 1983).

Some scholars try to avoid the problematic notion of *feature-sets* by assuming optimal-entity models. Whether Prototype models (Partee and Kamp 1995; Osherson and Smith 1981) or non-abstractionist Exemplar models (Brook 1987; Shanks and St. John 1994), the main idea in these theories is that a typicality degree is indicated by *degree of similarity to a representative entity*.

The problem in these theories is that similarity is, in many cases, measured by features. One can only categorize novel instances on the basis of their similarity to a known prototype or exemplar if there is some means of determining similarity, i.e. the connections that exist between the instances and the prototype or exemplar (Hampton 1997). And it is for this reason too, that, as we saw in 3.6, theories which stipulate prototypes or exemplars for each concept, without representing typicality features, fail to predict the connections that exist between the prototypes or exemplars of complex concepts, and the prototypes or exemplars of their constituents.

Finally, in eliminating the features from the analysis, the Supermodel Theory is silent with regard to the type of properties that speakers regard as typical of each predicate in a given context.

3.8 Conclusions of Part 3

The proposed measure functions fail to capture the fact that there exists a range of intermediate typicality degrees in denotation members. Hence, they fail to predict typicality in sharp predicates. This is a severe limitation, given that the most prominent examples of the prototype theory are indeed sharp predicates.

In addition, the theory fails to correctly represent the conjunction and sub-type effects, despite the use of two separate mechanisms, namely, the measure function and its modified version. Ideally, we would like to represent these effects correctly, and if possible, we would like one mechanism to derive both the conjunction and sub-type effects, i.e. typicality in basic and complex predicates.

We need an improved analysis, which, in addition to capturing the typicality effects in sharp and complex predicates, will capture the inherent context dependency of the typicality judgments and the gaps in these judgments. The analysis should leave context independent prototypes out. The status *prototypical* ought to be given to an entity only within a context (valuation).

Finally, the analysis ought to say exactly how *the weight of a feature* is determined and how *degrees in a feature* are determined, i.e. ***what*** a typicality degree ***is***. Ideally, the basic primitive in the analysis will be the typicality ordering (the denotation of *more / less typical than*). Numerical degrees will be derived from this ordering by some general strategy.

In the next part, I propose a new model which, it is argued, improves upon the previous analysis regarding precisely these points.

4. My Proposal: Learning Models

So what does a typicality-ordering stand for?

I believe this ordering is no more than a side effect of the order in which we learn that entities fall under a predicate, say, *bird.* We encode this learning order in memory, either during acquisition, or even as adults, within a particular context, when we need to determine which birds a speaker is actually referring to (the contextually relevant or appropriate set of birds).

4.1 Learning Models

Learning models represent information growth. More precisely, they represent the order in which entities are categorized under, say, *bird*, and *non-bird*. We start with a zero context, c_0, where denotations are empty, and from there on, each context is followed by contexts in which more entities are added to the denotations. In a total context t, every entity is either in the negative or in the positive denotation of each predicate.

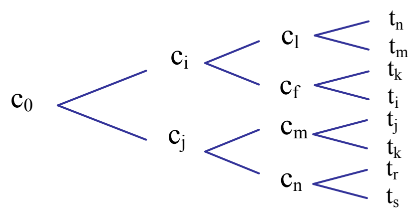
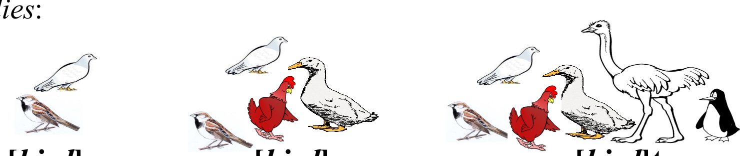


Figure 2: The contexts' structure in a Learning Model

For example, birdhood is normally determined first for *robins* and *pigeons*, later on for *chickens* and *geese*, and last for *ostriches* and *penguins*. Similarly, non-birdhood is detrmined earlier for *cows* than for *bats* or *butterflies*:



[*bird*]$\mathbf{c_0}$ **…** **[*bird*]**$\mathbf{c_i}$ **…** **[*bird*]**$\mathbf{c_n}$ **…** **[*bird*]**$\mathbf{t_s}$

Figure 3: An example of a branch in a Learning Model

Formally, I use the information structure called "Data Semantics" (Veltman 1984; Landman 1991). A learning model M* for a set of predicates A and domain D is a tuple **<C ,≤,**$\mathbf{c_0}$**,T>** such that:

[1] C is a set of partial contexts: in each c in C a predicate P is associated with partial positive and negative denotations: $\langle [P]^{+}_{c}, [P]^{-}_{c} \rangle$.

[2] ≤ is a partial order on C: $\forall P \in A$:

1. $\mathbf{c_0}$ is the minimal element in C under ≤: $[P]^{+}_{c0} = [P]^{-}_{c0} = \varnothing$
(Denotations are empty in c_0).

2. **T** is the set of maximal elements under ≤: $[P]^{+}_{t} \cup [P]^{-}_{t} = D$
(Denotations are maximal in T).

3. **Monotonicity**: $\forall c_1, c_2 \in C$, s.t. $c_1 \leq c_2$: $[P]^{+}_{c1} \subseteq [P]^{+}_{c2}$; $[P]^{-}_{c1} \subseteq [P]^{-}_{c2}$.

4. **Totality**: $\forall c \in C, \exists t \in T: c \leq t$ (Every c has some maximal extension t).

I also assume that in c, we consider as P, in addition to directly given Ps (i.e. members in $[P]^{+}_{c}$), also indirectly given Ps, i.e. entities whose P-hood can be inferred on the basis of the information in c (see *4.4.2* and 5.2). Formally, P-hood of an entity d can be inferred in c iff d belongs in $[P]^{+}_{t}$ in any t above c. I call this extended denotation *the super-denotation* of P:

5. "**Super-denotations**": $[P]_c = \cap\{[P]^{+}_{t} \mid t \in T, c \leq t\}$; $[\neg P]_c = \cap\{[P]^{-}_{t} \mid t \in T, c \leq t\}$

4.2 The typicality ordering

Given this basic ontology, I propose that we consider d_1 *more typical of P* than d_2 in a context t if and only if:

Either the P-hood of d_1 is established before the P-hood of d_2 (i.e. in a context that proceeds the context in which d_2 is added to the positive denotation),

Or the non-P-hood of d_2 is established before the non-P-hood of d_1 (i.e. in a context that proceeds the context in which d_1 is added to the negative denotation).

Formally, P's typicality ordering in t is the order in which entities are learnt to be P or ¬P in contexts under t:

[3] $\forall \mathbf{t} \in \mathbf{T}$: $(\langle \mathbf{d_1,d_2} \rangle \in [\leq \mathbf{P}]^+_{\mathbf{t}})$ if and only if:
$\forall c \leq t$: $(d_1 \in [P]_c \rightarrow d_2 \in [P]_c)$ & $(d_2 \in [\neg P]_c \rightarrow d_1 \in [\neg P]_c)$.
In any total t, d_1 is *equally or less (typical of) P* than d_2 iff
In any context c under t, if d_1 is P, d_2 is P, and if d_2 is ¬P, d_1 is ¬P.

Entity pairs might be added to ≤P in c either on the basis of direct pointing at them as standing in the relation *more typical of P*, or on the basis of indirect inferences from the rest of our knowlegde in c. That is, the extended typicality relation that holds between two entities in a partial context c can be formally defined using the supervaluation technique, as is usually done for propositions (Van Fraassen 1969):

$\forall \mathbf{c} \in \mathbf{C}$: $(\langle \mathbf{d_1,d_2} \rangle \in [\leq \mathbf{P}]_{\mathbf{c}})$ iff: $\forall t \geq c$: $(\langle d_1,d_2 \rangle \in [\leq P]^+_t)$
In any partial c, d_1 is *equally or less (typical of) P than* d_2 iff
In any total t above c, d_1 is *equally or less (typical of) P than* d_2.

Different ways to refer to ≤P differ in truth conditions. For instance, d_1 may be *more of a kibbutznik* but *less typical of a kibbutznik* than d_2 (if, say, d_2 has left the kibbutz but still looks and behaves like a kibbutznik). Yet, I believe that we need not pose different definitional constraints on *more P*, *more typical P* and *more relevant P*. The difference between these three comparative phrases is pragamatic in nature: It is generally assumed that the comparative *more P* makes use of a semantic ordering dimension in the meaning of P (Kamp 1995; Bartch 1984, 1986). Conversely, *more typical (of a) P* makes use of different, or additional, ordering properties, namely, criteria from world knowledge, not just semantic criteria. Finally, *relevant P* makes use of completely ad-hoc properties, not just world knowledge or semantic criteria. The effect of the ordering criteria on the ordering relation (and of the ordering relation on the ordering criteria) will be further discussed in 4.9-4.10. At this point, note only that, as desired, a possibly different ordering relation may be associated with a predicate in each context. This much context dependency is required in order to capture the typicality effects correctly (for further discussion of this point, see 4.8).

In the rest of part 4 we will see that a number of long-standing puzzles are now solved.

4.3 Deriving degrees

Numerical degrees are not directly given. The primitive notion is of ordering, which is more intuitive (cf. 3.5). However, numerical degrees can be derived easily, when needed, so that their ordering would conform to the typicality ordering.

For instance, assuming the facts in context t_s in Figure 3 above, and a small domain which consists of the six birds in the picture (a robin, a pigeon, a goose, a chicken, an ostrich and a penguin) and two non-birds (a butterfly and a cow), the robin would have degree 1 because everything, i.e., all 8 entities, is *equally or less typcal* than it. The goose would have degree 6/8 because only 6 of 8 entities are *equally or less typcal* than it, and so on.

Vagueness with regard to degrees (cf. 3.5) would be derived from gaps in the typicality ordering (see 4.8 below).

4.4 Intermediate typicality degrees for denotation members

4.4.1 Intermediate degrees

Recall that degrees of denotation members in Partee and Kamp's model were always maximal, i.e. 1. This is not the case in the current model. Rather, the earlier we learn that an entity is, e.g. a *bird*, the more typical we consider this entity to be.

Therefore, now we can account for the typicality effects in sharp predicates, which were problematic for Partee and Kamp. The typicality ordering, or graded membership effect, results from the fact that, in acquisition, or while disambiguating predicate meaning within a particular context, speakers encode different bird types in memory *gradually*. (Consider for a moment the predicate *prime number*. Despite its clear formal definition, the status of very big numbers with respect to *prime* is yet to be discovered by mathematicians!)

In Partee and Kamp 1995, the denotations of non-vague predicates (e.g. *bird*) are represented as total, Fregean entities, independent of speakers' experience or belief. But we already saw that typicality is connected to the set of entities which a speaker knows and considers relevant in a context (cf. 1; 3.4; 4.2). Moreover, the graded structure proposed in 4.2 does not interfere with the assumption that the denotation of *bird*, unlike the denotation of *chair*, though learnt gradually, is (normally) already fully specified in actual contexts of utterance. It is quite plausible to assume that it is already fully specified earlier in the context structure than the denotation of *chair* (which is more inherently vague). That is, the difference between vague and non-vague predicates (+/- Vague) is of quantity, more than of quality.

Finally, this intuitively felt difference between vague and non-vague predicates may have to do with other factors besides the level of vagueness in the denotation. Clearly, no speaker carries in mind an infinite list of all birds and non-birds. Crucially, an algorithm that enables speakers to determine the birdhood, or non-birdhood, of every new entity, can replace the assumption that the *bird* denotations are fully specified.

In 5.2 we discuss one such algorithm. We will see that the specification of only a few birds and a set of features allows speakers to automatically determine the birdhood of new items. The status of a novel item remains undetermined only if every known bird scores better than that item in the *bird* features, and that item scores better than every known non-bird in the *bird* features. However, this algorithm also applies to vague predicates like *chair*. Therefore, I would now like to draw attention to another algorithm, which, crucially, affects vague and non-vague predicates differently.

4.4.2 +/- Vague

Certain predicates such as *prime* or *chair* have a semantic necessary condition for membership. For example, the property *piece of furniture* is regarded as necessary for membership in $[chair]^{+}_{c}$ in a context of utterance c if and only if in every total context t extending c, every chair is a *piece of furniture*. Let the *PI* be a shorthand for the phrase: *positive integer that has no positive integer divisors other than 1 and itself*:

[4] The predicate *PI* is a *necessary condition* for membership in the denotation of *prime number* in a context c iff:
$\forall t \in T, t \geq c$: $[\textit{prime number}]^{+}_{t} \subseteq [\textit{PI}]^{+}_{t}$.

A predicate is a *semantic* necessary condition if and only if a competent speaker regards it as necessary in every context of utterance.

The difference between vague and non-vague (sharp) predicates is that *only in sharp predicates*, like *prime*, *the necessary condition can be treated also as a sufficient condition for membership and we may feel that we have a precisely defined denotation* (though in contexts this assumption of sufficiency may be dropped, when speakers refer to a more restricted set of relevant *prime* numbers).

The predicate *PI* is a *sufficient* condition for *prime numbers* in a context c iff:
$\forall t \in T, t \geq c$: $[\textit{PI}]^{+}_{t} \subseteq [\textit{prime number}]^{+}_{t}$

In contrast, with *chair,* the semantic necessary condition for membership, *piece of furniture*, definitely cannot be sufficient, since it doesn't distinguish chairs from other close sub-categories: *table*, *lamp* etc. Thus, predicates like *chair* are regarded as vague:

In most contexts of utterance c, a competent speaker regards the predicate *piece of furniture* as *necessary* for chairhood:
$\forall t \in T, t \geq c$: $[\textit{chair}]^{+}_{t} \subseteq [\textit{piece of furniture}]^{+}_{t}$.
But not as *sufficient* for chairhood:
$\neg\forall t \in T, t \geq c$: $[\textit{piece of furniture}]^{+}_{t} \subseteq [\textit{chair}]^{+}_{t}$.

Other predicates, such as *bald*, that do not have any semantic necessary condition for membership, are regarded as vague, too.

In sum, we saw that factors other than the level of vagueness in the denotation may be responsible for the intuitive distinction between vague and sharp predicates. We also saw that we are now able to correctly represent typicality in denotation members and sharp predicates. Next we will see that the second classical problem, i.e. *the conjunction fallacy* or *effect*, including its special sub-case – *the sub-type effect* (see 3.2-3.3), is also readily solved.

4.5 The sub-type effect

Sub-type effects can now be accounted for: The typicality degree of ostriches is greater in the predicate *ostrich* than in *bird*: if they are categorized late in *bird*, relative to other bird types, but early in *ostrich*, relative to other ostriches! Since this is a natural state of affairs, in most contexts typical ostriches are indeed considered as atypical *birds*.

For example, in the birds' model given in 4.3 above, the ostrich has a degree 2/8 in *bird*, because only 2 of 8 entities are *equally or less typical* than it in *bird*. Hence, it is *an atypical bird* in t_s. Yet, we can reasonably assume that this entity is the first member in the denotation of *ostrich* in t_s, i.e. its degree in *ostrich* is 1. Thus, it is both *an atypical bird* and *a very typical ostrich* in t_s.

4.6 The conjunction effect

Conjunction effects or fallacies are similarly accounted for: The degree of brown apples is greater in *brown-apples* than in *apple,* when they are categorized late under *apple*, relative to other apple-types (*red*, *green* etc.), but early under *brown apple*, relative to other brown apples.

Similarly, the typical *male-nurses* are atypical *males* when the earliest known *males* are not *nurses*. The typical *male-nurses* are also atypical *nurses* when the earliest known *nurses* are not *males*.

These facts fall into place without any new stipulations for complex predicates.

4.7 The negation effect

Negation effects are also accounted for without any new stipulations. The ordering of *non-bird* is, by the definition of a typicality ordering in 4.2, inverse to the ordering of *bird* in each context (for supporting evidence, see Smith et al 1988).

Exceptions to this generalization (cf. 2.1) are accounted for, since this inverse pattern is predicated only for the logical negation of a predicate. If a negated predicate like *non-bird* is contextually restriced to, say – *animals*, then it is not equivalent to the logical negation of *bird* and hence its ordering is not predicted to be inverse to the ordering of *bird*.

The third classical problem is the representation of partial and context dependent knowledge about typicality (see 3.4, 3.6). Let us see how the current proposal handles in these issues as well.

4.8 Partial knowledge

In a learning model, typicality degrees or relations may be unknown: A pair, say – a penguin and an ostrich, is in the gap of the ordering *more typical of a bird* in a context c, if it is still possible in c (i.e. true in some context following c) that the penguin is *more typical* in *bird*, and it is still possible that the ostrich is *more typical* in *bird*.

For example, if in context c_l in the learning model in Figure 2 (see 4.1), the penguin is already known to be a bird, but the ostrich is not yet known to be a bird, and in context c_f the ostrich is already known to be a bird but the penguin is not yet known to be a bird, then, in context c_i we do not yet know which bird is more typical, the penguin or the ostrich.

4.9 Context dependency

4.9.1 Context dependent ordering relations

The inherent context dependency of the typicality judgments is now predicted. Context independent (or valuation-independent) ordering relations are not part of the theory. As desired, the typicality ordering is defined per total context in the learning model.

But how is a contextual typicality ordering fixed? Context dependency in the interpretation of domains of quantifiers and conditionals is accounted for (Kadmon and Landman 1993; von Fintel 1994) by assuming that a set of properties restricts the domain to the set of relevant members in each context. Similarly, it is plausible that, within context, a set of properties (features) restricts predicate denotations to the set of relevant denotation members, those members, which the speaker is actually referring to (for a detailed discussion of the mechanism in which denotations are contextually restricted via properties, see Kadmon and Landman 1993; Sassoon 2002; and also 4.10 below).

Given this set of restricting features, the relevant typicality ordering of a predicate P in each context of utterance, is the ordering of *the conjunction of P and its restricting properties*. For example, *chickens* usually precede *robins* in being regarded as both *birds* and *walking in the barnyard*. Hence, their typicality degree in *bird* in the context of the utterance *birds walking* *in the barnyard* is predicted to exceed that of robins, as Roth and Shoben indeed found (see part 1).

4.9.2 Context dependent prototypes

Context independent (or valuation-independent) prototypes, in particular, are not part of the theory at all (cf. *2.3.1*, stipulation [5] in Partee and Kamp's model). In the current proposal, *in each context*, some entities are the best in each predicate: The earliest entities, among the available entities, which are known to be denotation members. In this way, we account for the ordering in *typical tall person* despite the fact that, out of context, there is no *maximal tallness*.

In addition, eliminating the prototypes from the theory considerably simplifies the taxonomy of predicates: The distinction between predicates without a prototype, predicates with a prototype that does not affect the denotation, and predicates with a prototype that affects the denotation (cf. *2.3.1*, stipulation [6]), is eliminated.

The intuitively felt differences between these predicate types is accounted for, again, in a quantitative rather than qualitative manner. These differences are induced by different extents of context dependency in the meaning of the predicate and its derived comparative. For example, in *taller,* the ordering criterion, and hence the ordering relation, is fixed semantically. But in *more typical of a tall person, player, tree* etc., *typical* associates more features with the predicate *tall* (context dependent ordering criteria). So the NP *typical tall person*, like *typical bird*, associates with a context dependent ordering relation. Such a context dependent ordering relation *must* be indicated by the operator *typical*.

4.9.3 *+/–Gradable, +/–Prototype*

Put more formally, +Gradable predicates, like *tall* and *bald*, (i.e. predicates that can directly combine with *more*) are distinguished from –Gradable predicates, like *bird* (that cannot combine with *more* unless modified by an operator like *typical*), in the following way:

Predicates like *bald* may not have a necessary condition for membership (cf. 4.4), but they do have a semantic ordering feature (see 4.10 for the definition of such a feature). Moreover, crucially, this ordering feature can be treated as *a necessary condition for membership in the derived comparative ≤bald* in a context of utterance c:

$\forall t \in T, t \geq c$: $[$*is more bald*$]^{+}_{t} \subseteq$ $[$*has less hair*$]^{+}_{t}$
i.e. if d_1 is *more bald* than d_2, then d_1 has *less hair* than d_2.

This single ordering feature can be treated also as *sufficient for membership in the ordering relation in c,* and hence, *we may feel that we have a precisely defined ordering relation*:

$\forall t \in T, t \geq c$: $[$*has less hair*$]^{+}_{t} \subseteq [$*is more bald*$]^{+}_{t}$

Other predicates, like *bird* or *prime*, do not have a single ordering feature: Out of context they have no semantic ordering criterion at all, and within contexts they are frequently associated with several ordering criteria (Kamp 1975). This can even happen with gradable adjectives like *bald* when, say – psychological features related to *baldness* are treated as ordering *bald* by typicality. In these contexts, *has less hair* cannot be treated as sufficient for membership in the ordering relation $\leq$*bald*, because one may be grasped as *balder* (or as *more typical of a bald person*) than other people with an *equal or greater amount of hair* (which nonetheless are psychologically more influenced by their baldness). When nothing is treated as necessary and sufficient for membership in the ordering relation, it remains vague and the predicate is felt to be –Gradable.

However, when a –Gradable predicate is associated with a set of ordering features, we do have partial knowledge regarding the ordering of entities. In particular, best cases can be identified: Those entities that satisfy all the ordering features are regarded as prototypes. Hence, predicates like *chair*, *bird* or *flu* are normally regarded as + Prototype.

This proposal predicts that a complex predicate would not be grasped as gradable even if its parts are gradable. In fact, such predicates do not combine with *more*:

7) * d_1 *is more midget giant than* d_2
8) * d_1 *is more fat bald than* d_2
9) * d_1 *is more clean tall than* d_2

They have two potential ordering criteria, so neither functions as sufficient for membership in their ordering relation. The appropriateness of *more P* seems to depend on the existence of a sufficient ordering criterion. In fact, even when P is sharp, *more P* improves whenever such a criterion becomes salient (e.g. *more pregnant*).

What about *multi-dimensional gradable predicates* such as *healthy*? These predicates seem to be misrepresented in the current proposal. They are felt to be +Gradable, not +Prototype (they directly combine with *more*), despite the fact that they are associated with a set of dimensions, not a single ordering dimension! For instance, one may be regarded as *healthy* if one is *generally healthy,* i.e. *healthy with respect to hair, heart, blood pressure, fever, skin* etc. None of the comparatives derived from these dimensions (nor the conjunction *healthier with respect to hair and healthier with respect to heart and…*) can be treated as *necessary and sufficient* for membership in the comparative ≤*healthy* (for example, one may be regarded *generally healthier* than others, while being *less healthy with respect to,* say, *the skin*). Yet, *healthy* can directly combine with *more*.

I believe that multi-dimensional gradable predicates like *healthy* are not associated with a set of ordering features in precisely the same way that +Prototype predicates, such as *bird,* are. In multi-dimensional gradable predicates we use (even explicitly), quantification over ordering dimensions, or respects (Bartch 1984; 1986): *generally healthy*, *healthy in every respect* etc. (i.e., a universal or generic quantifier ranges over the variety of ordering dimensions). The predicate is ordered by one dimension at a time. This is not the case with +Prototype –Gradable predicates like *bird*. Indeed, we do not usually say, or intend to say, that an entity is *generally a bird* or *a bird in every respect*.

4.10 Typicality features

Finally, the fourth classical problem, i.e. that of defining the notion of *a typicality feature* (or an ordering dimension), can now be dealt with. For each predicate P, speakers consider certain features as *typical of P*, e.g *feathers, small, flies* and *sings* are normally regarded as *typical of birds*. In addition, it is common in Philosophy and Psychology to assume that each feature is assigned a weight, and generally, the typicality degree of say, a robin in *bird*, is indicated by the weighted-mean of its degrees in all the bird features: How well it scores in *flies, sings, small* etc. However, scholars still cannot tell the exact conditions under which a property is regarded as a typicality feature and they hardly agree about how a weight of a feature is determined.

4.10.1 Ceteris paribus correlation

Having stated what a typicality ordering is (cf. 4.2), we can now state that a property like *flying* or *being-small* counts as a typicality feature of a predicate like *bird* iff the ordering in the feature correlates with the ordering in *bird* ***ceteris paribus*** i.e.:

[5] Any entity more typical in *flying* than other entities, **and not less typical in other features like *small***, is more typical of a *bird*.

Exceptions (items which are more typical in *flying* but less typical in *bird* or vice versa) are allowed when (and only when) the ordering in two bird-features (e.g. *flying* and *small*) is inverse.

4.10.2 Feature weights

Given this generalization, we can now state that the greater the overlap between the typicality ordering (the set of entity pairs where the former entity is more typical than the latter entity) of a feature and the typicality ordering of *bird*, the higher the feature's weight, i.e. the more central it is considered in ordering birds. Formally, the weight of a typicality feature F is indicated by the extent of overlap between (or the relative size of the intersection of) its orderings, ≤F, and P's ordering, ≤P:

[6] ***The weight of F in P*** : = $|([\leq F]_t \cap [\leq P]_t)|/|(D\times D)|$

For example, the ordering of *bird* and of *small* (which in the context of *bird* means *a robin-sized-bird*) are identical with only few exceptions, so this feature's weight is significant. It plays a central role in ordering entities by typicality in *bird*:

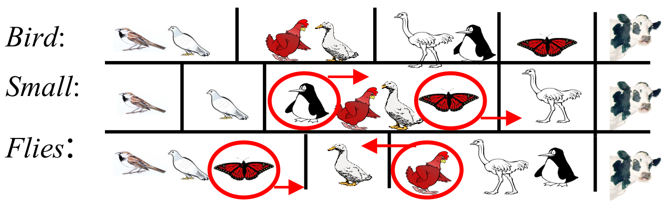


Figure 4: High overlap between the typicality-ordering of *bird* and of *small / flies* (Birds in the same block are, roughly, equally typical); Exceptions are marked in red circles.

However, a property might exist, like – *animal,* with an ordering which correlates ceteris paribus with the ordering in *bird* as required, i.e. any entity more typical in *animal* than other entities, *and not less typical in other bird features*, is more typical of a *bird*. However, the overlap between the ordering of *bird* and that of *animal* is poor, since many typical animals are atypical birds (most of them are actually not birds at all). Therefore, the feature weight of *animal* is not significant.

We can now assume that the set of predicates in our language also consists of (in addition to 'normal' predicates, which denote sets of individuals) predicates of the form: *a typical feature of P*. These predicates denote sets of features. The denotations of these predicates grow gradually through contexts, just like any other predicate denotation (for a detailed discussion of a model with such feature sets, see Sassoon 2002).

5. What exactly do Learning Models model? More findings

In part 4, we saw that, by assuming that the typicality ordering is no more than a partial order, which stands for the order in which entities are learnt to be members or non-members in a denotation, we shed light on a variety of typicality effects which are traditionally regarded as puzzling.

However, two more clarifications with regard to the concept "learning order" are required. Both have to do with the fact that the learning order as it is encoded in memory is not always equivalent to the actual temporal order in which items are added to the denotation, due to two factors.

5.1 Corrections

The first factor has to do with our ability to make corrections in our knowledge. *What if my initial exposure to birds was through ostriches*?? Initially, I would think that ostriches are representative birds. Later on, I would have to correct my beliefs. Formally, I would jump to a different branch in the context-structure, where ostriches are indeed represented as less typical than other birds. Indeed, it is known that first exposure to an atypical item slows down acquisition (Mervis & Pani 1980). Why? Because learners induce wrong category features: For instance, in our example, wrong optimal size, *running* instead of *flying* etc.

5.2 Inferences: Indirect learning

The second factor has to do with indirect learning, i.e. with our ability to add items to the denotation even if they were never given to us as such. We can infer the membership of certain new items by using the knowledge given to us already by the known denotation members and features. I assume that – if one has knowledge about the bird features (unlike the children in the experiments of Mervis & Pani 1980, just cited) – then new, previously unavailable entities, which are better than known birds in the *bird*-features, once they become available, are automatically regarded as birds, too (otherwise rule [5] in *4.10.1* will be violated; Sassoon 2002). So we have *a learning algorithm* which overcomes arbitrary gaps in our learning order. For example, categorization of, say – a *chicken* or a *goose*, in *bird* – implies the bird-hood of anything more typical than a *chicken* or a *goose*, like a *duck*, once it is available. Indeed, it is also known that previously unavailable typical instances are frequently (falsely) assumed to be known: (Reed 1988). Why? Given their high scores in the typicality features, they should already be known denotation members!

But, not so for atypical ones. For example, if the known birds are *robins, pigeons, geese* and *chickens*, in the exposure to *ostriches* we would not infer their bird-hood automatically. They would remain in the gap because it is still possible that they diverge too much from the known birds. Hence, they are regarded as less typical.

An intruiguing evidence for indirect learning of this sort was found in a study of aphasic patients by Kiran & Thompson, which was based on previous findings in neural network simulations. These studies demonstrate that exposure to a whole range of atypical items *and features* results in spontaneous recovery of categorization of untrained more typical items, but not vice versa.. That is, the membership of more typical instances can be indirectly automatically inferred from the membership of less typical instances, but not vice versa, as predicted.

5.3 Conclusions of part 5

Initially, direct learning of the category membership of certain entities occurs, and possibly also direct learning of certain typicality features. The order of learning the category members is encoded in memory. Then, this ordering is enriched and corrected, based on indirect inferences. If the learning-order of a property highly correlates with the category learning-order, this property is treated as a typicality feature, too. In addition, in the exposure to new entities, more entities are added to the denotation. If the new entities score highly in the typicality features, corrections in the learning order are made, such that these entities are encoded as typical. In this way, speakers overcome the effects of arbitrary gaps in their learning order.

6. Conclusions

In addition to the coupling between typicality and learning (which is demonstrated by a range of studies), learning models capture a wide range of typicality effects which were long-standing puzzles. These puzzles include the typicality effects in sharp and complex predicates (in particular the conjunction effect /fallacy), the context dependency and partiality of the knowledge about the typicality relations and degrees, and the definition of a feature and a feature weight.

Unlike previous theories (fuzzy models or supermodels), the current proposal predicts the typicality effects in complex predicates without any new stipulations for the purpose, i.e. without a complement rule for negated predicates and a minimal degree rule (cf. 2.1) or a modified membership function (cf. *2.3.2*) for modified nouns.

By insisting on a highly context dependent representation for the typicality ordering, a number of theoretical entities are eliminated from the analysis, among which are the context independent prototypes and the measure functions.

The coupling between typicality and membership is captured via the gradual learning of the denotation members. This spares us the need to stipulate two separate sets of values for the membership function and the typicality function, and renders the theory more elegant.

In addition, the taxonomy of predicate types is drastically simplified. The intuitively felt differences between predicate types are accounted for using the (well-defined) notions of *ordering features*, of *necessary and sufficent conditions for membership,* and of *partial ordering relations*. Unlike the measure function over sets of valuations, these notions are psychologically real: There is abundant evidence that speakers associate predicates with partial sets of ordering relations, ordering features, and necessary conditions for membership.

Given the elegance and the wide array of predictions of the learning model, it seems that our understanding of the typicality effects and their relation to predicate meaning, has considerably improved.

References

Aarts, Bas, David Denison, Evelien Keizer, and Gergana Popova (eds.)
2004 *Fuzzy Grammar, a Reader*. Oxford University Press.
Armstrong, Lee, Lila Gleitman, and Henry Gleitman
1983 What some concepts might not be. *Cognition* 13: 263-308.
Bartsch, Renate,
1986 Context dependent interpretations of lexical items, J. Groenendijk, D. de Jongh, M. Stokhof (eds.) *Foundations of Pragmatics and Lexical semantics*, GRASS 7, Foris, Dordrecht.
1984 The structure of word meanings: Polysemy, Metaphor, Metonymy. In: Landman Fred & Veltman Frank (Eds.), *Varieties of Formal semantics*, GRASS 3, Foris, Dordrecht.
Barsalou, Lawrence
1983 Ad hoc categories. *Memory and Cognition 11*: 211-227.
Batting W.F. and Montague W.E.,
1969 Category Norms for Verbal Items in 56 Categories. *Journal of Experimental Psychology* Monograph 80 (3) Pt. 2.
Brooks, L.R.
1987 Nonanalytic cognition In U. Neisser (Eds.), *Concepts and Conceptual Development: Ecological and intellectual factors in categorization*, 141-74, Cambridge University Press.
Costello, Fintan
2000 An exemplar model of classification in simple and combined categories. In: *Proceedings of the Twenty-Second Annual Conference of the Cognitive Science society*, Lila.Gleitman and K. Joshi (eds.), 95-100. Mahwah, N. J.: Erlbaum.
Dayal, Veneeta,
2004 Number Marking and (In)definiteness in Kind Terms. *Linguistics and Philosophy* 27(4): 393 – 450.
Fein, Kieth,
1975 Truth, Vagueness and Logics, *Synthese* 30: 265-300.
Gardenfors, Peter
2004 *Conceptual Spaces, The Geometry of Thought*. MIT Press.
Hampton, James
1997 Conceptual Combination. In: *Knowledge, Concepts and Categories*, Koen Lamberts and David Shanks (eds.), 135-162. Cambridge, MA: The MIT Press.
Heit, Evan
1997 Knowledge and concept learning. In: *Knowledge, Concepts and Categories*, Koen Lamberts and David Shanks (eds.), 135-162. Cambridge, MA: The MIT Press.
Kadmon, Nirit, and Fred Landman,
1993 Any. *Linguistics And Philosophy* 16: 353-422.

Kamp, Hans
1975 Two theories about Adjectives. In: Edward Keenan (ed.), *Formal Semanticvs for Natural Language*.
Keil, Frank
1987 Conceptual development and category structure. In: *Concepts And Conceptual Development,* Ulrich Neisser (ed). Cambridge University Press.
Kiran, Swathi and Cynthia Thompson,
2003 The role of semantic complexity in treatment of naming deficits: Training categories in fluent aphasia by controlling exemplar typicality. *Journal of Speech Language and Hearing Research* 46: 608-622.
Klein, Ewan.
1980 A semantics for positive and comparative adjectives. Linguistics and Philosophy 4:1–45.
Lakoff, George
1973 Hedges: a study in meaning criteria and the logic of fuzzy concepts. *Journal of Philosophical logic* 2: 458-508.
1987 *Women, Fire and Dangerous Things: What Categories Reveal about the Mind*. Chicago University Press.
Landau, Barbara
1982 Will the real grandmother please stand up? The psychological reality of dual meaning representations. *Journal of Psycholinguistic Research* 11(1): 47-62.
Landman, Fred
1991 *Structures For Semantics*. Dordrecht: Kluwer Academic Publishers.
Lynott, Dermot, and Michael Ramscar
2001 Can we model conceptual Combination using distributional Information, *12th Irish Conference on Artificial Intelligence and Cognitive Science 5.9-7.9*.
Mervis, Carolyn, and Eleanor Rosch
1981 Categorization of natural objects. *Annual review of psychology* 32: 89-115.
Mervis, Carolyn, and John Pani
1980 Acquisition of basic object categories. *Cognitive Psychology* 12: 496-522.
Murphy, Gregory, and Douglas Medin
1985 The role of theories in conceptual coherence. *Psychological Review* 92(3): 289-316.

Murphy, Gregory, and Mary Lassaline
1997 Hierarchical structure in concepts and the basic level of categorization. In: *Knowledge, Concepts and Categories,* Koen Lamberts and David Shjanks (eds.), 93-131.Cambridge, MA: The MIT press.
Murphy and Smith
1982 Basic level superiority in picture categorization. *Jurnal of Verbal Learning and Verbal Behaviour* 21: 1-20.
Osherson, Daniel, and Edward Smith
1981 On the adequacy of prototype theory as a theory of concepts. *Cognition* 11: 237-262.
Osherson, D.N. And E.E. Smith, O.Wilkkie, A.Lopez, E.Shafir,
1990 Category Based Induction" *Psychological Review* 97: 185-200.
Partee, Barbara, and Hans Kamp
1995 Prototype theory and compositionality. *Cognition* 57: 129-91.
Reed, Stephen
1988 *Cognition, Theory and Applications*. Pacific Grove: Brooks/Cole.
Rips, Lance
1975 Inductive judgments about natural categories. *Jurnal of Verbal Learning and Verbal Behaviour* 14: 665-81.
Rosch, Eleanor
1973 On the internal structure of perceptual and semantic categories. In: *Cognitive Development and the Acquisition of Language*, T. E. Moore (ed.), New York: Academic Press.
Roth Emilie and Edward Shoben
1983 The effect of context on the structure of categories. *Cognitive Psychology* 15: 346-378.
Smith, Edward, Daniel Osherson, Lance Rips, Margaret Keane
1988 Combining prototypes, a selective modification model. *Cognitive Science* 12: 485-527.
Sassoon, Galit
2002 Semantics with clusters of properties. In: *Proceedings Of Israel Association For Theoretical Linguistics 18*, Yehuda Falk (ed.), Bar Ilan University.
2004 Review of: Aarts, Bas; Denison, David; Keizer, Evelien; Popova, Gergana (2004) *Fuzzy Grammar, a Reader*, Oxford University Press. *The linguist list:* http://linguistlist.org/issues/15/15-3335.html.
2005 The notion of typicality. In: *Proceedings of From Gram to Mind*, Mai 19-21, Bordeaux (www.post.tau.ac.il/~gala).
Shanks, David, and M.F St. John
1994 Characteristics of dissociable human learning systems. *Behavioral and Brain Sciences* 17(3): 367-447.

Tversky, Amos, and Danniel Kahneman
1983 Extensional versus intuitive reasoning: the conjunction fallacy in probability judgments. *Psychological Review* 90: 293-315.

van Fraassen, B.C.,
1969 Presuppositions, Supervaluations and Free Logic. In: K. Lambert (Ed.), *The Logical Way of Doing Things*: 67-91. New Haven: Yale University Press.

Veltman, Frank
1984 Data Semantics. In: *Truth, Interpretation and Information Proceedings Of The 3rd Amsterdam Colloquium*, Jeroen Groenendijk, Theo Janssen and Martin Stokhof (eds.)

von Fintel, Kai,
1994 *Restrictions on Quantifier Domains*. PhD Dissertation, Amherst: University of Massachusetts.

Wittgenstein, Ludwig
1968 Reprint: *Philosophical Investigations*, Translated By G.E.M Anscombe, 3rd edition, Oxford: Blackwell. Original edition, 1953.

Zadeh, Lofti
1965 Fuzzy sets, *Information and control* 8: 338-53.